

Phy 230 B Lecture 7

massless spin 1 particles

Fock Space: $|0\rangle \quad \hat{a}_\pm(p)|0\rangle = |p, \pm\rangle \dots \quad p^2=0$

$$\hat{U}(\Lambda)|p, \pm\rangle = e^{\mp i(\mathbf{H})(\Lambda, p)} |p, \pm\rangle$$

$$\underbrace{\langle 0 | \hat{A}^\mu(x) | p, \pm \rangle}_{\neq 0} = \epsilon_\pm^\mu(p) e^{-ip \cdot x} \quad 4 \neq 2$$

translation invariance

(interpolating field)

$$\langle 0 | \underbrace{\hat{U}(\Lambda)^\dagger}_{\text{Lorentz Trans of field}} \cdot \hat{A}^\mu(x) \cdot \underbrace{\hat{U}(\Lambda)}_{\text{Lorentz Trans of state}} | p, \pm \rangle =$$

assuming $\hat{A}^\mu = \Lambda^\mu_\nu \hat{A}^\nu(\Lambda^{-1}x)$ (is going to fail)

$$\Lambda^\mu_\nu \epsilon_\pm^\nu(p) \stackrel{?}{=} e^{\mp i(\mathbf{H})(\Lambda, p)} \epsilon_\pm^\mu(\Lambda p)$$

claim: ~~$\epsilon_\pm^\mu(p)$~~

$$K^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad \epsilon_\pm^\mu(k) = ?$$

$$\Lambda^\mu_\nu = W^\mu_\nu = \begin{pmatrix} -i(\mathbf{E} \cdot \mathbf{T} + \mathbf{E}^2 T^2) & -i(\mathbf{H} \cdot \mathbf{T}) \\ & \end{pmatrix}^\mu_\nu$$

$W \cdot K = K$ analog of rotation for rest frame in massless case

$\Lambda \rightarrow W$
 $p \rightarrow K$

$$\rightarrow W^\mu_\nu \epsilon^\nu(K) = e^{\mp i(\mathbf{H})(\mathbf{H})} \epsilon_\pm^\mu(K)$$

$$-i(\mathbf{E} \cdot \mathbf{T} + \mathbf{E}^2 T^2 + (\mathbf{H} \cdot \mathbf{T}))^\mu_\nu \epsilon_\pm^\nu(K) = \mp i(\mathbf{H}) \epsilon_\pm^\mu(K)$$

$\uparrow V(\mathbf{H}), \mathbf{E}^{1,2}$

$$\Rightarrow \begin{cases} (T^{1,2})^\mu_\nu \epsilon_\pm^\nu(K) = 0 \\ (J^3)^\mu_\nu \epsilon_\pm^\nu(K) = \mp \epsilon_\pm^\mu(K) \end{cases}$$

$\leftarrow$ H.W. to count equations (secretly ~~17~~ due to arbitrary rescaling of ϵ)
8 unknowns

$(T^{1,2})^\mu_\nu \epsilon_\pm^\nu(K) = 0 \Rightarrow \epsilon_\pm^\mu(K) \propto \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \propto K^\mu$ claim this is the only equation

$$(J^3)^\mu_\nu \epsilon_\pm^\nu(K) = \mp \epsilon_\pm^\mu(K) \Rightarrow \epsilon_\pm^\mu(K) \propto \begin{pmatrix} 0 \\ \mp 1 \\ 0 \\ 0 \end{pmatrix}$$

Allow $\epsilon_{\pm}^{\mu}(k)$ = linear combination of $\begin{pmatrix} 1 \\ 0 \\ 0 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ i \end{pmatrix}$
 3 polarization vectors

Allow $\tilde{\epsilon}_{\pm}^{\mu}(k) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix} + \alpha_{\pm} k^{\mu}$

$$W(\mathbb{H}, \Lambda) = e^{-i(\Xi^1 T^1 + \Xi^2 T^2)} e^{-i\mathbb{H} J^3}$$

$$W(\mathbb{H}, \Lambda)^{\mu}_{\nu} \epsilon_{\pm}^{\nu}(k) = e^{i\mathbb{H}} \epsilon_{\pm}^{\mu}(k) + \left[\alpha_{\pm} + e^{i\mathbb{H}} \frac{\Xi^2 - i\Xi^1}{\sqrt{2}E} \right] k^{\mu}$$

Want

$$\hat{A}^{\mu}(x) = \int (dp) \sum_{h=\pm 1, 0} \left[\hat{a}_{\pm}(p) \epsilon_{\pm}^{\mu}(p) e^{-ip \cdot x} + h.c. \right]$$

$$\hat{U}(\Lambda)^{\dagger} \hat{A}^{\mu}(x) \hat{U}(\Lambda) = \Lambda^{\mu}_{\nu} \hat{A}^{\nu}(\Lambda^{-1}x) + \dots$$

Equations of motion: direct consequence

$$\begin{cases} \partial_{\mu} \hat{A}^{\mu}(x) = 0 \leftarrow p_{\mu} \epsilon_{\pm}^{\mu}(p) = 0 \\ \square \hat{A}^{\mu}(x) = 0 \leftarrow p^2 = 0 \end{cases}$$

$$\Rightarrow \partial_{\mu} \hat{F}^{\mu\nu}(x) = 0$$

Classical Gauge Theory

* gauge "symmetry": $A_{\mu} \mapsto A_{\mu} + \partial_{\mu} \Omega$
 invariance $F_{\mu\nu} \mapsto F_{\mu\nu}$

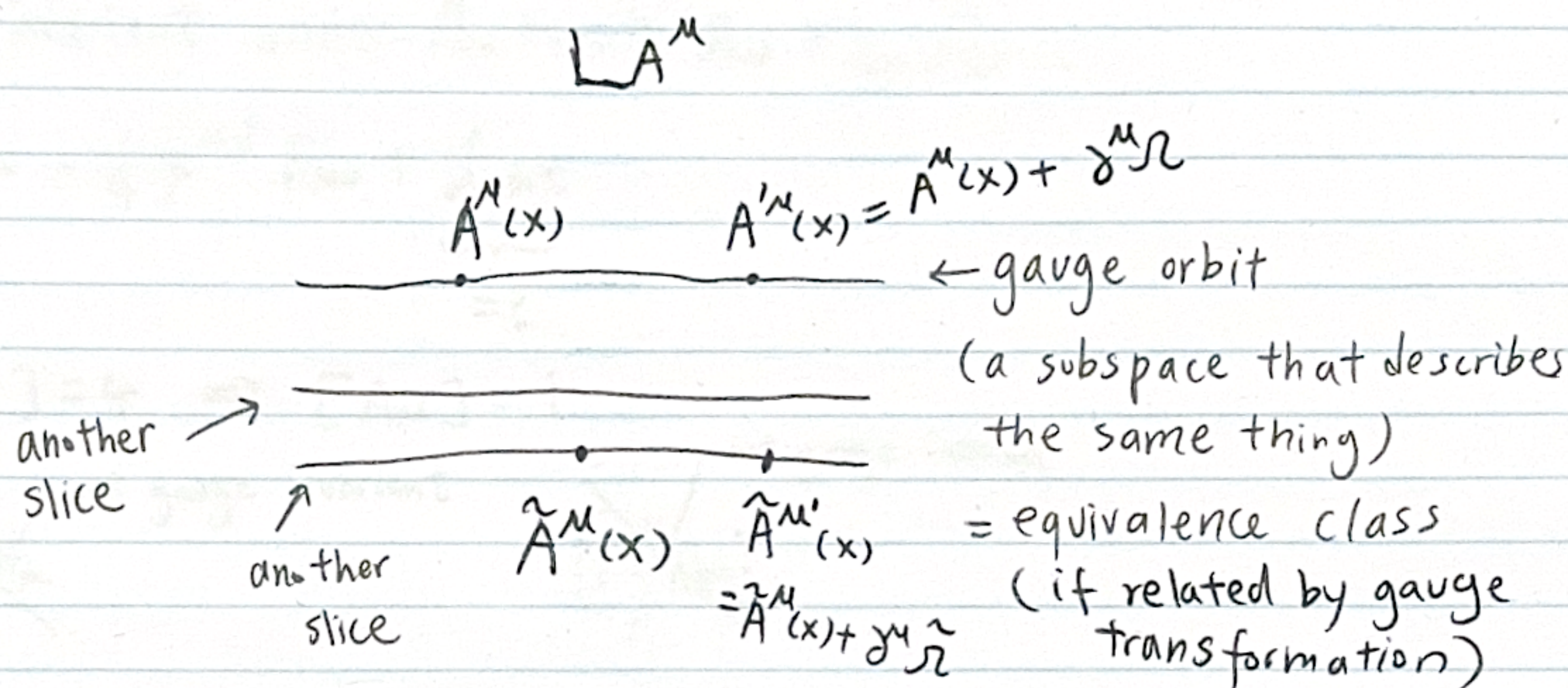
$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad \left(\text{also } \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right) = \partial_{\mu} K^{\mu}$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$\delta \mathcal{L} = 0 \Rightarrow \partial_{\mu} F^{\mu\nu} = 0$$

Idea: $A^{\mu}(x) \sim A'^{\mu}(x) = A^{\mu}(x) + \partial^{\mu} \underbrace{\Omega(x)}_{\text{arbitrary}}$

"gauge principle": A^μ, A'^μ are physically indistinguishable



"Gauge Fixing": choose a representative of each equivalence class

e.g.: impose "gauge condition" that chooses 1 guy in ~~each~~ ^{each} eqv. class.

Example: $\partial_\mu A^\mu = 0$ ← Lorentz Gauge is Lorentz invariant

$$A'^\mu = A^\mu + \partial^\mu \Omega \quad \uparrow \quad !!! \neq \text{Lorentz}$$

$$\partial_\mu A'^\mu = \partial_\mu A^\mu + \square \Omega$$

Example: $A^0(x) \equiv 0$ ← Weyl Gauge (temporal gauge)

Setting time component to zero

claim: $\hat{A}^0(x) = 0$ $[\epsilon_\pm^0(k) = 0 \Rightarrow \epsilon_\pm^0(p) = 0]$

Recap: If we want $\hat{A}^\mu(x)$ to be interpolating field for massless spin 1
 $\Rightarrow \hat{A}^\mu(x) \neq 4\text{-vector field}$

$\hat{A}^0(x) \equiv 0$??? really weird

* Interpreting: $A^\mu(x) \sim A'^\mu(x) = A^\mu(x) + \partial^\mu \Omega(x)$ for some $\Omega(x)$

$A^0(x) = 0$ gauge choice

Gauge Invariant Interactions?

$\mathcal{L}$ invariant under $A_\mu \mapsto A_\mu + \partial_\mu \Omega$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \underbrace{\mathcal{L}_{int}}_{=?}$$

$$[\mathcal{L}] = 4 \Rightarrow [A_\mu] = 1$$

~~$A^\mu A_\mu$~~ not gauge invariant

~~$(A^\mu A_\mu)^2$~~ not gauge invariant

$\mathcal{L}_{int}$ function of $F_{\mu\nu}$

$$\underbrace{F^{\mu\nu}}_{\substack{\uparrow \\ \text{anti} \\ \text{symmetric} \\ \text{tensor}}} \underbrace{F_{\nu\rho} F^\rho_\mu}_{\substack{\text{symmetric} \\ \text{under } \mu \leftrightarrow \nu}} \equiv 0$$

anti symmetric tensor

$$F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} F_{\sigma\mu} \neq 0$$

$$(F^{\mu\nu} F_{\mu\nu})^2$$

$\vdots$

photons $\rightarrow$

$$\mathcal{L}_{int} \sim \frac{1}{M^4} (F_{\mu\nu})^4$$

irrelevant interactions

relevant marginal vs.

irrelevant interactions

Add scalars, fermions

$$\underbrace{[\bar{\psi}\psi]}_{[]=3} \underbrace{F^{\mu\nu} F_{\mu\nu}}_{[]=4} = 7 \text{ also irrelevant!}$$

require fermion fields transform under gauge transformations.